Journal of Engineering Physics and Thermophysics, Vol. 83, No. 4, 2010

CALCULATION OF THE KINETICS OF DRYING DISPERSE MATERIALS ON THE BASIS OF ANALYTICAL METHODS

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UDC 621.928.9:66.099.2:66.046

In the context of a systematic approach, the general problem of the kinetics of drying disperse materials is split into two basic levels: the micro- and macrokinetic ones, and in conformity with this, the problems of mathematical modeling of the kinetics of drying on the micro- and macrolevels are considered. The merits and drawbacks of the application of a moving and fixed coordinate systems for calculating the kinetics of drying of disperse materials in continuously operating apparatuses are discussed. A number of kinetic mathematical models of the process of drying disperse materials in its second period are presented and analyzed.

Keywords: *kinetics, dispersity, heat and mass transfer, moving and fixed coordinates, inhomogeneity of particles with respect to their size and time of residence in an apparatus.*

Introduction. Drying of disperse materials is widely used in chemical, food, microbiological, light, and woodworking industries, in thermal engineering, and in agricultural production. The central and most complex part of the calculation of the process of drying is the kinetic calculation whose aim, when calculating a construction, is the determination of the overall dimensions of a continuously operating apparatus that would ensure the assigned productivity, whereas in calculation of batch equipment — finding the duration of the main stage of the process — the stage of drying.

In the kinetic calculation of driers, a large number of methods are used which can be subdivided into the following three groups: empirical, semiempirical, and mathematical (analytical and numerical); their analysis is given in [1]. A gradual departure from various empirical and semiempirical methods of kinetic calculation of driers is observed whose basic merit was their simplicity, and the main drawback leading to the problem of scale transition was the inadequate accuracy and reliability of calculations in transfering the results of laboratory research to industrial facilities. The present time witnesses increasingly wider introduction of mathematical (analytical and numerical) methods into engineering practice. This is attributed to the development of the theory, accumulation of data on heat and mass transfer coefficients, development of software, refinement of computational methods, and the widespread application of PCs. The greatest difficulties in the calculation of the kinetics of drying are caused by its second period (the period of the falling rate of drying) precisely which is considered below.

1. General Strategy of Constructing Kinetic Models of Drying. The mathematical modeling of drying is needed for correct organization of the process, determination of energy expenditures on its realization, calculation, and selection of the primary (a drier) and auxiliary equipment. The central part of such modeling is the description of the kinetics. For the kinetic calculation of driers a great number of methods are applied which in [1] are subdivided into empirical, semiempirical, and theoretical (analytical and numerical). The aim of this work is to present developments (of the present author and of other researchers) on mathematical modeling of the process of drying of disperse materials and on kinetic calculation of driers on the basis of analytical models.

Two different approaches to constructing mathematical models of the kinetics of drying of disperse materials in continuously operating apparatuses are possible. The first is based on the use of the moving (Lagrangian) coordinate system connected with the center of some particle moving in the apparatus. The second employs the fixed (Eulerian) coordinate system fixed on the apparatus body.

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The first approach presupposes a description of the kinetics of drying of single particles with subsequent generalization of this description to the entire ensemble of moving particles. The second approach does not consider individual particles — it considers all the particles moving in the apparatus.

In using both approaches, for the convenience of calculation and increasing its accuracy, it is advisable that when applying the strategy of a system analysis [2], the general kinetic problem that describes the process of drying a material in a drier be decomposed into two basic levels: 1) microkinetic one (description of the kinetics of drying of single granules, particles, bodies — in the first approach or of a differentially small volume of material in a drier — in the second approach) and 2) macrokinetic one — description of the process of drying a material in the whole working volume of the apparatus. The advantage of the decomposition of the general kinetic problem to the micro- and macrokinetic levels is that at the microlevel in the experimental determination of the curves of drying and heating particles in a monolayer, their shape, anisotropicity, and other specific factors typical of the material being dried are taken into account correctly automatically, whereas at the macrolevel the possibility appears of more accurately reflecting the influence exerted on the kinetics by the conditions of the process organization that are connected with the characteristic features of the apparatus construction and of the interaction of phases in it. This allows one to more judiciously go over from the results of laboratory investigations to calculation of industrial apparatuses, which lessens the problem of scale transition [3]. Since the description of the kinetics of the second period of drying causes the greatest difficulty, it is this period that the present analysis is devoted to.

2. Description of the Process in a Moving Coordinate System. ***2.1. Microkinetics of drying.*** The first approach requires the description of the microkinetics of the drying of single particles. Physically this can be done most judiciously on the basis of the A. V. Luikov system of interrelated differential heat- and mass transfer equations [4]:

$$\partial u/\partial\tau = \text{div}\,[k\,(u, t)\ \text{grad}\ u + \delta_t\,(u, t)\ \text{grad}\ t]\ ; \tag{1}$$

$$c\,(u, t)\,\rho_0\,(\partial t/\partial\tau) = \text{div}\,[\lambda\,(u, t)\ \text{grad}\ t] + \varepsilon^* r^* \rho_0\,(\partial u/\partial\tau)\ . \tag{2}$$

Equations (1) and (2) contain the thermophysical characteristics k, δ_t, c, λ, and ε, which, however, are known completely only rarely. Additional difficulties in calculation are introduced by their dependence on moisture content and temperature, whereas a consideration of this factor is needed for ensuring the required accuracy of calculations. In the case of high-temperature processes the system of equations (1) and (2) is supplemented by a differential equation for the field of pressures that describes molar transfer of moisture in the material under the action of the general pressure gradient.

The greatest difficulty in the calculation of the microkinetics of drying of capillary-porous and colloid capillary-porous materials by this model is caused by the assignment of a numerical value of the criterion of internal phase transformations ε^*, whose value changes along the particle coordinate and in the course of drying. To overcome the difficulty in the assignment of ε^*, it is appropriate to make one of the following assumptions in engineering calculation of the temperature field of a particle: 1) evaporation occurs from the body surface (in the case of nonporous [5] and microcapillary-porous ($r_{\text{gov}} \le 10^7$ m) materials; 2) the heat sink is uniform over the entire volume of the particle, which is applicable for capillary-porous materials with macrocapillaries; 3) in the course of drying there occurs deepening of the evaporation front (this model is first of all valid for hydrophobic materials with poorly wetted pore walls, as well as for materials with a homogeneous distribution of pores in which capillary transfer of a liquid-phase moisture is difficult). The models of the kinetics of heating corresponding to the first two variants are given in [5], whereas the model for the third variant was developed in [6, 7]. It should be emphasized that here we are dealing only with the localization of the surface or of the evaporation zone in the heat conduction problem. As to the mass conduction equation, the coefficient of mass conduction k [5] (according to A. V. Luikov, the coefficient of moisture diffusion in a material [4]) integrally reflects the actual transfer of moisture in the material in the form of both a liquid and a vapor in each section of the material, i.e., these assumptions are not extended to mass conduction. Analytical solutions of nonlinear equations of heat and mass transfer for bodies of conical shape are absent, but in applying the zonal method of calculation for engineering purposes one can employ the solutions of differential equations of heat and mass transfer with constant coefficients of transfer, which makes it possible to allow for a change in the parameters of the process and in the thermophysical characteristics in the course of drying [5].

We will briefly analyze each of the above-named microkinetic models connected with the assumptions about internal phase conversions.

1) The model presupposing evaporation from the body surface. As noted above, the calculation of the microkinetics of drying on the basis of heat- and mass conduction equations (1) and (2) is generally problematic because of the above-noted difficulties. The problem is significantly simplified in the case of deep drying of granulated polymers due to the specificity of the process. Investigations show [5] that this process has some specific features: 1) the internal moisture is removed extremely slowly, and the process is entirely controlled by internal diffusion; b) because of the low intensity of drying, a granule is rapidly heated, and its drying proceeds under the conditions of practically equal temperatures of the material and of the drying agent in its vicinity; c) according to the technological requirements the material must be dried up to a very low moisture content (0.01–0.02%), which requires an accurate assignment of equilibrium moisture content and which becomes one of the basic parameters in the description of microkinetics; d) because of the low value of the temperature gradients in a granule the heat and moisture conductivities are negligibly small; e) the granules have a rather regular shape, admitting the statement of the analytical problem of mass conduction.

In the case considered the description of the microkinetics of drying can be done on the basis of the following one-dimensional problem:

$$\partial u/\partial\tau = \operatorname{div}\,(k\,(u,t)\,\operatorname{grad} u)\,, \quad 0 < x < R\,, \quad \tau > 0\,; \tag{3}$$

$$u\,(x,\tau) = u_{\text{in}}\,(x)\,, \quad 0 \le x \le R\,, \quad \tau = 0\,; \tag{4}$$

$$u\,(x,\tau) = u_{\text{eq}}\,, \quad x = R\,, \quad \tau > 0\,; \tag{5}$$

$$\partial u\,(x,\tau)/\partial x = 0\,, \quad x = 0\,, \quad \tau > 0\,. \tag{6}$$

In the case of deep drying of polymer granules, mass transfer in them follows the mechanism of molecular diffusion [5]; therefore in Eq. (3) $k \equiv D_{\text{ef}}$, with $D_{\text{ef}} = f(u, t)$, i.e., Eq. (3) is nonlinear. Since its analytical solution for bodies of canonical shape (plate, cylinder, sphere) is absent, for engineering calculations we use the zonal method, the essence of which is that the entire range of change of the moisture content of the material subjected to drying is subdivided into a number of concentration zones, in each of which the coefficient D_{ef} and equilibrium moisture content u_{eq} are assumed constant, and the calculation of the duration of drying reduces to the solution of the corresponding linear differential equation of diffusion for the symmetric problem which, at a constant boundary condition in application to the mean volumetric moisture content in a granule, has the form

$$\bar{E}_i = \sum_{n=1}^{\infty} B_n \exp\,(-\,\mu_n^2 \text{Fo}_{\text{m}})\,, \tag{7}$$

where $B_n = f(\text{Bi}_{\text{m}})$ is the pre-exponential factors in the solutions of the problems of mass conduction for bodies of an appropriate shape and μ_n are the characteristic roots of the solutions of problems [5].

Using the theorem of the mutual multiplication of solutions, it is possible to generalize Eq. (7) to the case of granules in the form of a rectangular parallelepiped and a restricted cylinder, and for a regular regime of mass transfer to obtain the dependence for calculating the needed time of residence of a granule in the ith concentration zone:

$$\tau_i = \frac{1}{k_i \sum_{j=1}^{s} \frac{\mu_j^2}{R_j^2}} \ln \frac{\prod_{j=1}^{s} B_{j,i}}{\bar{E}_i}\,, \quad i = 1, 2, ..., n\,; \quad j = 1, ..., s\,, \tag{8}$$

where at $i = 1$ the coefficient $B_{j,i}$ is equal to the pre-exponential factor in the solution of the one-dimensional problem of diffusion for the jth coordinate, whereas at $i > 1$ the coefficient $B_{j,i}$ can be taken equal to unity [5]; $\overline{E}_i = (\overline{u}_{f,i} - u_{eq,i})/(\overline{u}_{in,i} - u_{eq,i})$ is the relative moisture content of the granule in the ith concentration zone; $\mu_{i,j}^2$ are the roots of characteristic equations of the solutions of diffusion problems at the boundary condition of the first kind; $s = 1, 2, 3$ respectfully for granules in the form of a sphere, finite cylinder, and rectangular parallelepiped, and R_j is the radius of a sphere or of a cylinder, and half of the plate thickness.

In the case of convective drying of microcapillary-porous materials at not very high temperatures at which the temperature of the material $t_{mat} < 100^{o}C$, the thermal moisture conductivity can be neglected, and the process of moisture transfer in an isotropic particle of a regular geometric shape can be described on the basis of Eq. (3) and, for calculation of its heating, to use the solution of the differential equation of heat conduction provided, as noted above, that evaporation occurs at the particle surface. The problem of heating a particle can be formulated in this case as

$$c (u, t) \rho_0 (\partial t/\partial \tau) = \mathrm{div} [\lambda (u, t) \,\mathrm{grad}\, t] , \quad 0 < x < R , \quad \tau > 0 ; \tag{9}$$

$$t (x, \tau) = t_{in} (x) , \quad 0 \le x \le R , \quad \tau = 0 ; \tag{10}$$

$$\lambda \left(\frac{\partial t}{\partial n}\right)\bigg|_{sur} = \alpha (t_{dr} - t_{sur}) - r^{*} i (\tau) , \tag{11}$$

$$\partial t (x, \tau)/\partial x = 0 , \quad x = 0 , \quad \tau > 0 . \tag{12}$$

In the linear statement of the problem of mass transfer, the intensity of drying $i(\tau)$ can be found from the solution of the problem of mass conduction (7) in the form

$$i (\tau) = R_v \rho_0 (u_{in} - u_{eq}) \frac{a}{R^2} \mathrm{Lu} \sum_{n=1}^{\infty} \mu_n^2 B_n \exp (- \mu_n^2 \mathrm{Fo_m}) . \tag{13}$$

Solving the problem (9)–(12) subject to Eq. (13) at c, λ, r^{*}, k, and α = const, we obtain the following dependence for calculating the average volume temperature of the particle:

$$\overline{\Theta} = \frac{\overline{t} (\tau) - t_{dr}}{t_{in} - t_{dr}} = \sum_{k=1}^{\infty} B_k$$

$$\times \left[\exp (- \mu_k^2 \mathrm{Fo}) + \frac{R_v}{R} \mathrm{Ko\, Lu} \frac{\mu_k^2}{\mathrm{Bi}} \sum_{n=1}^{\infty} B_n \frac{\mu_n^2}{\mu_k^2 - \mathrm{Lu}\, \mu_n^2} \left[\exp (- \mu_n^2 \mathrm{Lu\, Fo}) - \exp (- \mu_k^2 \mathrm{Fo}) \right] \right] . \tag{14}$$

When $\mathrm{Bi_m} \to \infty$, Eq. (14) takes the form

$$\overline{\Theta} = \frac{\overline{t} (\tau) - t_{dr}}{t_{in} - t_{dr}} = \sum_{k=1}^{\infty} B_k$$

$$\times \left[\exp (- \mu_k^2 \mathrm{Fo}) + \frac{R_v}{R} \mathrm{Ko\, Lu} \frac{\mu_k^2}{\mathrm{Bi}} \sum_{n=1}^{\infty} \frac{b}{\mu_k^2 - \mathrm{Lu}\, \mu_n^2} \left[\exp (- \mu_n^2 \mathrm{Lu\, Fo}) - \exp (- \mu_k^2 \mathrm{Fo}) \right] \right] , \tag{15}$$

where b = 2 for a plate, b = 4 for a cylinder, and b = 6 for a sphere.

2) Model postulating a uniform heat sink from the entire volume of the particle. In this case, in Eq. (2) the criterion of internal phase conversions $\varepsilon^* = 1$. To obtain the microkinetic computational relation, we rewrite this equation in a linear statement as

$$\partial t/\partial\tau = a\Delta t + r^*\rho_0 (\partial u/\partial\tau) , \tag{16}$$

with the boundary condition for it

$$\lambda \left(\frac{\partial t}{\partial n}\right)_{\rm sur} = \alpha \,(t_{\rm dr} - t_{\rm sur}) . \tag{17}$$

In zonal calculation of the kinetics of drying, the solution of Eq. (16) at a uniform initial condition, boundary condition (17), and a symmetric problem in the regular regime of heat transfer for the volume-average temperature in the ith concentration zone can be presented as

$$\overline{\Theta}_i = \frac{t_{{\rm dr},i} - \overline{t}_i}{t_{{\rm dr},i} - \overline{t}_{{\rm in},i}} = \frac{1}{A}\,{\rm Po}_i \left(1 + \frac{\Gamma}{{\rm Bi}_i}\right) - \left(1 - \frac{{\rm Po}_i}{\mu_{1,i}^2}\right) B_{1,i} \exp\,(-\,\mu_{1,i}^2 {\rm Fo}_i) , \tag{18}$$

where A = 3, 8, 15; Γ = 3, 4, 5 for a particle in the form of an infinite plate, infinite cylinder, and a sphere, respectfully; $B_{1,i}$ is the pre-exponential factor in the solution of the heat conduction problem for the first term of the series; $\mu_{1,i}$ is the first root of the characteristic equation in the solution of the heat conduction problem.

In the Pomerantsev number in Eq. (18), for engineering purposes the derivative $\partial u/\partial\tau$ in each ith concentration zone is approximated by the linear function $(\Delta\overline{u}/\Delta\tau)_i$.

3) The model foreseeing the deepening of the evaporation surface in the course of drying. This model was developed in [6] in application to an infinite plate, and in [7], to an infinite cylinder. Many particles that compose a solid phase in the process of drying have the form of plates or fibers, and therefore they can be likened to bodies of this shape. The physical notion on the deepening of the evaporation front leads to the statement of the Stefan problem with a moving boundary. In particular, the mathematical statement of the problem of heating a cylindrical fiber in the process of drying with a mobile phase-change boundary is as follows:

$$\frac{\partial t_1(r,\tau)}{\partial\tau} = a\,\frac{\partial^2 t_1(r,\tau)}{\partial r^2} + \frac{1}{r}\,\frac{\partial t_1(r,\tau)}{\partial r} , \quad \tau > 0 , \quad y(\tau) < r < R ; \tag{19}$$

$$\lambda\,\frac{\partial t_1(r,\tau)}{\partial r} = \alpha\,(t_{\rm dr} - t_1(r,\tau)) , \quad r = R , \ \tau > 0 ; \tag{20}$$

$$t_1(y(\tau),\tau) = t_2(y(\tau),\tau) , \quad \tau > 0 ; \tag{21}$$

$$\lambda\,\frac{\partial t_1(y(\tau),\tau)}{\partial r} + \varepsilon\rho_{\rm w} r^*\,\frac{\partial y}{\partial\tau} = 0 , \quad \tau > 0 ; \tag{22}$$

$$-\,\varepsilon\rho_{\rm w}\,\frac{\partial y}{\partial\tau}\bigg|_{y=y(\tau)} = K_p\,(p_{\rm sat} - p_{\rm dr}) , \quad \tau > 0 ; \tag{23}$$

$$y(0) = R , \quad \tau = 0 ; \tag{24}$$

$$t_2(r, \tau) = f(r), \quad 0 \le r \le y(\tau). \tag{25}$$

The problem (19)–(25) was solved in [7] analytically using the method of differential series [8]. The model allows one to determine the temperature field of a dried layer $t_1(r)$ in the region $y(\tau) < r < R$ and the law of the movement of the evaporation boundary $y = y(\tau)$ and thus to calculate the microkinetics of the drying of a particle. An approximate solution can be found with any degree of accuracy.

The advantage gained by the description of the microkinetics of drying on the basis of the Luikov differential equations is that there exists a possibility of calculating not only the change in the average moisture content of a particle in time, but also the field of moisture contents in it, which is also of interest in many cases. Moreover, in parallel, one collects information on the kinetics of heating a particle, which is important for thermolabile materials, and on passage to the macrolevel, makes it possible to calculate the material temperature at the exit from a drier and thus to more accurately compose its thermal balance.

In the case where for describing the microkinetics it is difficult to apply the solution of interrelated differential equations of internal heat and mass transfer (because of the complex shape of particles or absence of data on internal heat- and mass transfer coefficients), in the second period of drying one can use for the purpose a modified equation of mass transfer over the solid phase which for single particle can be written in the form

$$-d\bar{u}/d\tau = K_{\rm v}(\bar{u} - u_{\rm eq}), \tag{26}$$

where $K_{\rm v}$ is the kinetic coefficient having the meaning of the volume coefficient of mass transfer over the solid phase (at $K_{\rm v}$ = const this is the coefficient of drying K according to Sherwood, when the rectifying line of the rate of drying on the graph demonstraing the dependence of the rate of drying on the moisture content of the material is drawn into the point of critical moisture content or, according to Luikov, when it is drawn into the point of reduced critical moisture content [4]). The advantage of Eq. (26) is that it automatically accounts for the shape of particles and their anisotropicity and polydispersity (if this equation is used not for individual particles, but for their certain collection), and the drawback is that the coefficient $K_{\rm v}$ is the regime one (and not a reference quantity in contrast, for example, to the coefficient of moisture diffusion in a polymer granule $D_{\rm ef}$). By its physical essence Eq. (26) is semiempirical.

2.2. Macrokinetics of drying. At the macrokinetic level account is taken of the change in the process parameters over the working volume of an apparatus. Macrokinetic models include equations of the material and heat balance, equations describing the hydrodynamics of flows, and some other accompanying factors inherent in one or other process of drying. An example of constructing a mathematical macrokinetic model based on the use of the Luikov interrelated mass- and heat transfer equations (1) and (2) can be work [9] that considered a continuous-band convection drier. The characteristic feature of the process is that there was a crossflow of phases in the drier and their ideal displacement and that, in describing the microkinetics, the thermal moisture conductivity was considered negligibly small. The calculation of the process was carried out by the zonal analytical method. Contrastingly, in [10] a macrokinetic mathematical model of drying of a disperse material in a continuously operating drier was developed which was also based on Eqs. (1) and (2), but in application to counterflow motion of phases.

In many processes, when a moving coordinate system is used in describing the process with micro- to-macrolevel transition, it is necessary to take into account the inhomogeneity in the size of particles and in the time of their residence in an apparatus. These factors are essential for the accuracy of kinetic calculation and for the uniformity of drying, i.e., for the quality of the product dried. The influence of the indicated inhomogeneities on the average moisture content of a material $\bar{\bar{u}}$ at the exit of particles from a continuously operating apparatus can be taken into account using the equation

$$\bar{\bar{u}} = \int_{R_{\min}}^{R_{\max}} f(R) \int_0^{\infty} f(\tau)\, \bar{u}(R, \tau)\, dR d\tau, \tag{27}$$

where $f(R)$ and $f(\tau)$ are the differential functions of the distribution of particles with respect to the size and time of residence in an apparatus.

TABLE 1. Error of Kinetic Calculation of the Process of Deep Convective Drying of Granulated Polymers without Account for Their Polydispersity [11]*

Polymer material	Form of granule	$\delta_{\Sigma\Psi}$, %
Polyethylene terephthalate of brand A	Finite plate	15.24
Polyamide P-610L	Finite cylinder	
Polyamide P-610L-SV-30	»	
Polyamide P-12É	»	
Polypropylene	»	19.3
Polystyrene UPM-0703L	»	
SFD VM-BS	»	
Polycarbonate "Diflon" casted	»	
Polystyrene "Styron"	»	
Polystyrene PSM-115	»	
ABS-plastics 1210	Sphere	31.95

*$\delta_{\Sigma\Psi}$, relative error due to disregard of the polydispersity of granules over its entire sizes.

In [11], the factual polydispersity of differently produced polymer granules was investigated experimentally, and its influence on the macrokinetics of the deep drying of these materials was analyzed. Calculations showed that the inhomogeneity in the size of particles exerts a substantial influence on the kinetics of the process (see Table 1) and, consequently, must be taken into account in calculations.

The data on $\delta_{\Sigma\Psi}$ listed in the table make it possible to approximately account for the influence of the actual polydispersity of polymer granules on the their required residence time in a drier not invoking calculations by integral relation (27). For this purpose, first from Eq. (8), using the zonal method, one calculates the needed residence time, in an apparatus $\bar{\tau}_{\sigma_\psi} = 0$, of monodisperse granules with average sizes $\bar{R}_1$, $\bar{R}_2$ and $\bar{R}_3$ (here for spherical particles $\bar{R}_1$ is their average radius, $\bar{R}_2$ and $\bar{R}_3$ are absent; for particles in the form of a finite cylinder $\bar{R}_1$ is their average radius, $\bar{R}_2$ is the mean one-half of their length; $\bar{R}_3$ is absent; for particles in the form of a rectangular parallelepiped $\bar{R}_1$, $\bar{R}_2$, and $\bar{R}_3$ are mean halves of their sizes along the three coordinate axes). Then, depending on a concrete polymer, from Table 1 one selects the correction $\delta_{\Sigma\Psi}$ and verifies the needed residence time of granules in an apparatus $\bar{\tau}_{\sigma_\psi}$ under the condition of their polydispersity corresponding to a certain value of the overall dispersion of granules $\sigma_{\Sigma\Psi}$ over the sizes R_1, R_2, and R_3:

$$\bar{\tau}_{\sigma_\psi} = \bar{\tau}_{\sigma_\psi = 0}\,(1 + \delta_{\Sigma\Psi})\,, \tag{28}$$

where $\bar{\tau}_{\sigma_\psi}$ is the needed residence time of polydisperse granules in an apparatus; $\sigma_\psi = \sigma_R/\bar{R}$; the subscript $\Sigma\Psi$ means that the dispersion over all the sizes of the particle is taken into account [11].

The intensity of longitudinal mixing of a solid phase is characterized by the dispersion $\sigma^2_{\theta\text{sol}}$; with application of the diffusional model of longitudinal mixing [12] its coupling with the corresponding number $\text{Pe}_{\text{long.sol}}$ is established by the relation [5]

$$\sigma^2_{\theta,\text{sol}} = \frac{2}{\text{Pe}_{\text{long.sol}}} - \frac{2}{\text{Pe}^2_{\text{long.sol}}} + \frac{2}{\text{Pe}^2_{\text{long.sol}}} \exp\left(-\text{Pe}_{\text{long.sol}}\right)\,, \tag{29}$$

where $\sigma_\theta = \sigma_\tau/\bar{\tau}$.

The influence of the longitudinal mixing of a solid phase on the kinetics of continuous drying of granulated polymers was studied in [11, 13, 14]. The necessity of its consideration in the kinetic calculation of the investigated constructions of apparatuses was shown, constructive measures for improving the hydrodynamics of the solid phase flow that would decrease the intensity of longitudinal mixing, as well as the dependences for calculating the number $\text{Pe}_{\text{long.sol}}$ were suggested. On the whole, at the present time a small number of experimental data on the longitudinal mixing of a solid phase in continuously operating driers of different types are known, which complicates the microto-macrolevel transition with the use of relation (27). Investigation of longitudinal mixing in laboratory setups with transfer of the data obtained to an industrial object is of low efficiency because of the inability to adequately model the hydrodynamic conditions of flow in laboratory conditions [3], whereas experiments on industrial facilities, if such are available, are difficult because of the absence, as a rule, of the possibility to vary input parameters and put a tracer into a flow.

3. Description of the Process in a Fixed Coordinate System. ***3.1. Microkinetics of drying.*** In a fixed coordinate system, for the description of a steady-state process of mass transfer in a continuously operating convection drier, ordinary differential equations are used, which simplifies the mathematical model. The microkinetics here can be expressed on the basis of Eq. (26) which in this case takes the form

$$-d\bar{\bar{u}}/d\tau = K_{\mathrm{v}}\,(\bar{\bar{u}} - u_{\mathrm{eq}})\,. \tag{30}$$

Various other empirical and semiempirical microkinetic equations are also used [1].

3.2. Macrokinetics of drying. In constructing a mathematical model for an entire apparatus, allowance for the hydrodynamic structure of flow is needed. In the case of a nonideal structure of flow, its approximate description on the basis of diffusional and cellular models, models with recycling, bypass, and their combinations is constructive. In particular, the use of one-parameter diffusion model in combination with Eq. (30) leads to the following statement of a macrokinetic problem:

$$v_{\mathrm{sol}}\frac{d\bar{\bar{u}}}{dx} = D_{\mathrm{long.sol}}\frac{d^2\bar{\bar{u}}}{dx^2} - K\,(\bar{\bar{u}} - u_{\mathrm{eq}})\,,\quad 0 < x < l\,; \tag{31}$$

$$\bar{\bar{u}}\,(x) = u_{\mathrm{in}} + \frac{D_{\mathrm{long.sol}}}{v_{\mathrm{sol}}}\frac{d\bar{\bar{u}}}{dx}\,,\quad x = 0\,; \tag{32}$$

$$\frac{d\bar{\bar{u}}}{dx} = 0\,,\quad x \to \infty\,. \tag{33}$$

Condition (32) describes a jumpwise change in the moisture content of a material at its entry into a drier because of the mechanism of longitudinal mixing involved in the operation. In the formulation of the given problem, it is assumed that the derivative $d\bar{\bar{u}}/dx = 0$ for $x \to \infty$, which corresponds to the notion of achievement of an equilibrium moisture content on infinitely long contact of the material with a drying agent. This distinguishes the given statement of the problem from that presented in [15–17], where the given derivative was taken equal to zero at $x = l$. Experimental data point to the fact that at the end of the operating zone of the drier, i.e., at $x = l$, the derivative $d\bar{\bar{u}}/dx \neq 0$ at $x = l$ [18].

At K, v_{sol}, u_{in}, and $D_{\mathrm{long.sol}}$ = const the solution of problem (31)–(33) has the form

$$\bar{\bar{E}} = \frac{\mathrm{Pe}_{\mathrm{long.sol}}}{\mathrm{Pe}_{\mathrm{long.sol}} - m}\exp\,(mz)\,, \tag{34}$$

where

$$m = \frac{\mathrm{Pe}_{\mathrm{long.sol}} - \sqrt{\mathrm{Pe}_{\mathrm{long.sol}}\,(\mathrm{Pe}_{\mathrm{long.sol}} + 4\gamma)}}{2}\,,\quad m < 0\,. \tag{35}$$

Figure 1 illustrates the influence of the $\mathrm{Pe}_{\mathrm{long.sol}}$ number of the kinetics of drying of a disperse material in a continuously acting drier, and Fig. 2, the influence of the kinetic parameter γ on the kinetics of drying. As is seen from Fig. 1, with decrease in the number $\mathrm{Pe}_{\mathrm{long.sol}}$, i.e., with increase of the effect of longitudinal mixing, the current

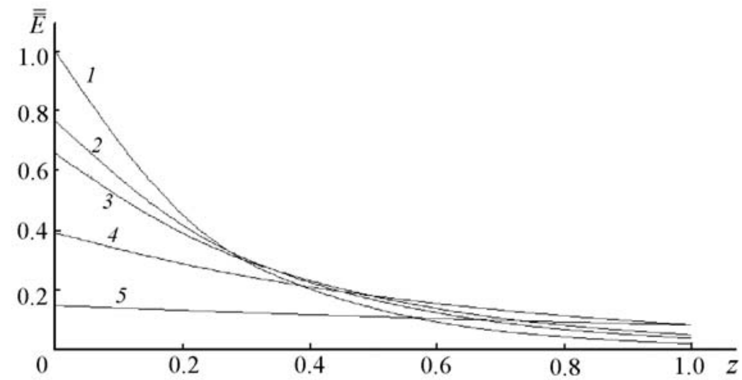

Fig. 1. Dependence $\bar{\bar{E}} = f(z)_{\text{Pe}_{\text{long.sol}}}$ at $\gamma = 4$: 1) $\text{Pe}_{\text{long.sol}} \to \infty$; 2) $\text{Pe}_{\text{long.sol}} =$ 10; 3) 5; 4) 1; 5) 0.1.

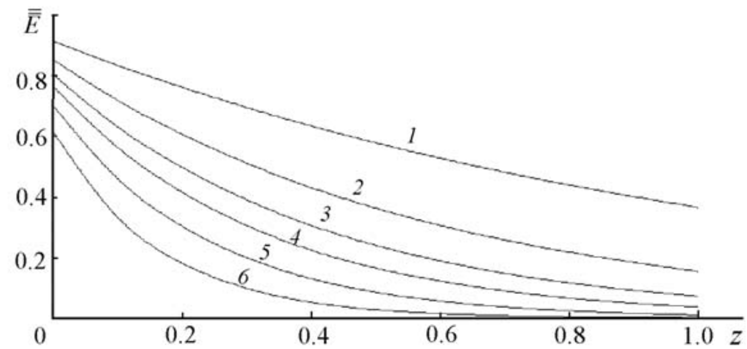

Fig. 2. Dependence $\bar{\bar{E}} = f(z)_\gamma$ at $\text{Pe}_{\text{long.sol}} = 10$; 1) $\gamma = 1$; 2) 2; 3) 3; 4) 4; 5) 6; 6) 10.

relative moisture content of the material along the solid phase flow is equalized and approaches the finite one, whereas the finite moisture content increases because of the decrease in the average moving force of the process (difference in the moisture contents of the material: the factual and equilibrium ones). At entry of a material into an apparatus there occurs a jumpwise decrease in the moisture content of the material because of the involvement of the mechanism of longitudinal mixing in conformity with boundary condition (32). With increase in the value of the dimensionless parameter γ, which is directly proportional to the coefficient K, the drying naturally proceeds more intensely.

A comparison of the given computational relations with the use of moving and fixed coordinate systems shows that in the second case the mathematical description is simpler, since the stationary process is considered. However, in this case the experimental values of the coefficient of mass transfer involved, which are determined in laboratory investigations and by their character, are regime quantities. In transferring these data to industrial objects the problem of scale transition usually arises [3]. However, in the case of application of a moving coordinate system, the modeling of the process can be performed on the basis of the thermophysical characteristics of the particles of the material dried, which are reference data, and methodically this is more justifiable.

Conclusions. The performed analysis shows that the analytical methods of kinetic calculation of driers are being successfully developed with the use of both approaches for describing the kinetics — both on the basis of a moving and fixed coordinate systems each of which has its advantages and disadvantages. The application of the zonal method makes it possible to use analytical methods of kinetic calculation in engineering practice and to avoid extra difficulties connected with the application of nonlinear differential equations of transfer, simultaneously ensuring an accuracy sufficient for practical purposes. Models appeared that describe not only purely drying processes, but also various combined processes (drying and granulation [19], sublimation and crushing [20], drying and crushing [21], etc.). Simplified hydrodynamic models of flow structures make it possible to take into account the longitudinal mixing of phases, the phenomena of recycling and bypass not invoking complex hydrodynamic calculations; however, their use is hindered by the absence of corresponding experimental data that identify the parameters of these models.

NOTATION

$A_{eq} = u_{eq}/C_{dr}$, coefficient of distribution of the function of concentration phase equilibrium, kg/(kg of dry material)/(kg/m^3); a, thermal diffusivity of material, m^2/s; $Bi = \alpha R/\lambda$, thermal Biot number; $Bi_m = (\beta_{dr} R)/(k\rho_0 A_{eq})$, mass transfer Biot number; C_{dr}, vapor concentration in a drying agent, kg/m^3; c, heat capacity of material, J/(kg·K); D_{long}, coefficient of longitudinal diffusion (of longitudinal mixing), m^2/s; D_{ef}, effective coefficient of moisture diffusion in a polymer granule, m^2/s; $\bar{\bar{E}} = \dfrac{\bar{\bar{u}}(x) - u_{eq}}{u_{in} - u_{eq}}$, relative moisture content average over all the particles; $Fo = a\tau/R^2$, thermal Fourier number; $Fo_m = k\tau/R^2$, mass transfer Fourier number; i, intensity of drying of a single particle, kg/(m^2·s; K, coefficient of drying, s^{-1}; K_{eq}, mass transfer coefficient related to partial vapor pressure, kg/(m^2·s·Pa); K_v, volumetric coefficient of mass transfer over a solid phase, s^{-1}; $Ko = r^*(u_{in} - u_{eq})/[c(t_{dr} - t_{in})]$, Kossovich number; k, mass conductivity coefficient, m^2/s; $Lu = a/k$, Luikov number; l, length of the solid phase flow in an apparatus, m; n, number of concentration zones in zonal calculation; $Pe_{long.sol} = v_{sol} l/D_{long.sol}$, Peclet number of the longitudinal mixing of a solid phase; $Po_i = \left[\rho_0 r^* R^2 \left(\dfrac{\Delta\bar{u}}{\Delta\tau}\right)_i\right] \Big/ \left[\lambda_i\left(t_{dr,i} - \bar{t}_{in,i}\right)\right]$, Pomerantsev number in the ith concentration zone; p, partial vapor pressure, Pa; R, half the plate thickness, radius of a cylinder or sphere, m; $\bar{R}$, average value of R (mathematical expectation), m; R_v, ratio of the volume of a particle (or of the volume of many particles) to its (their) surface, m; r^*, vapor generation heat, including the heat of moisture desorbtion, J/kg; r_{gov}, governing radius of pores, m; t, temperature, ^{o}C; $\bar{t}$, volume-average temperature of a particle; u, $\bar{u}$, $\bar{\bar{u}}$, local moisture content, moisture content average over the volume of a particle, and that average over many particles, respectively, kg/(kg of dry material); v, velocity, m/s; x, spatial coordinate (the Cartesian coordinate for a plate or solid phase flow in an apparatus, radial for a cylinder and sphere), m; $z = x/l$, relative coordinate; α, heat transfer coefficient, W/(m^2·K); β_{dr}, coefficient of mass transfer related to the difference of concentrations C, m/s; $\gamma = Kl/v_{sol}$, dimensionless kinetic parameter; δ_t, relative coefficient of heat and moisture conductivity, 1/K; λ, thermal conductivity of a material, W/(m·K); ε, porosity of a particle, m^3/m^3; ε^*, criterion of internal phase transformations; ρ, density, kg/m^3; σ_R, dispersion of particles over their size R; σ_τ, dispersion of particles over their residence time in an apparatus τ; τ, time, s; $\bar{\tau}$, average (flow rate) residence time of material in an apparatus. Subscripts: dr, drying agent; ef, effective; eq, equilibrium; f, finite; gov, governing; i, number of concentration zone in zonal calculation of the kinetics of drying; in, initial; j, number of coordinate in the solution of mass conduction problem; k, number of the term of exponential series in the solution of the heat conduction problem; long, longitudinal diffusion (longitudinal mixing); m, mass transfer; mat, material; n, number of the term of exponential series in the solution of the mass conduction problem; s, number of coordinates in the solution of mass conduction problem; sat, saturation; sol, solid phase; sur, near the particle surface; t, heat and moisture conduction; w, water; ∇; volumetric; Ψ, relative dispersion over sizes R; 0, absolutely dry material; 1, dried region in a particle; 2, wet region in a particle.

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